The effect of cross flow can be a significant reduction of the penetration of a particle, especially if the initial longitudinal velocity difference, $u_g - u_{p_o}$, exceeds the lateral injection velocity.

As an illustrative example, consider a 100-μm particle having a density of 2.5 g/cm^3 injected laterally ($u_{p_o} = 0$) with a velocity of 10 m/sec into an air flow of 20 m/sec and atmospheric density ($\rho_g = 1.2 \times 10^{-3}$ g/cm^3, $\mu = 1.8 \times 10^{-4} P$). For these data, the Reynolds number based on v_{p_o} is 67 and $\alpha = 4.7$. Equation (12) yields $Y_{st} = 77$ cm and Fig. 1 shows that $Y/Y_{st} = 0.3$. The actual penetration therefore is only 23 cm. Figure 2 indicates that $Y/Y_{u_g - u_{p_o}=0} = 0.735$; penetration without cross flow therefore would be 31 cm.

Calculation of particle penetration often is based on Stokes drag because of the resultant simplification of the equations. As the foregoing example shows, penetrations computed in this manner may be much too large because Stokes drag is too low and the effect of cross flow is neglected. The present analysis provides a convenient method for the calculation of particle penetration without having to resort to the unrealistic assumption of Stokes drag.

Particles injected by a carrier gas are directly affected by the cross flow only after their inertia carries them into the main flow. The early part of the "free-particle" trajectory therefore is influenced by the carrier gas. This effect can be expected to be of little importance for large particles but may be significant for small particles. Proper assessment of this problem requires knowledge of the behavior of gas-particle jets in a cross flow. Such studies are currently in progress.

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Transient Molecular Concentration Measurements in Turbulent Flows Using Rayleigh Light Scattering

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Introduction

THE current trend in turbulence research is away from a preoccupation with simple time averages and towards the investigation of detailed time histories of the flow variables. This follows from the recognition that many turbulent flows are dominated by relatively well-ordered structures. Vitally important to this development is the ability to make reliable time and space-resolved measurements of composition and temperature in the turbulent field.

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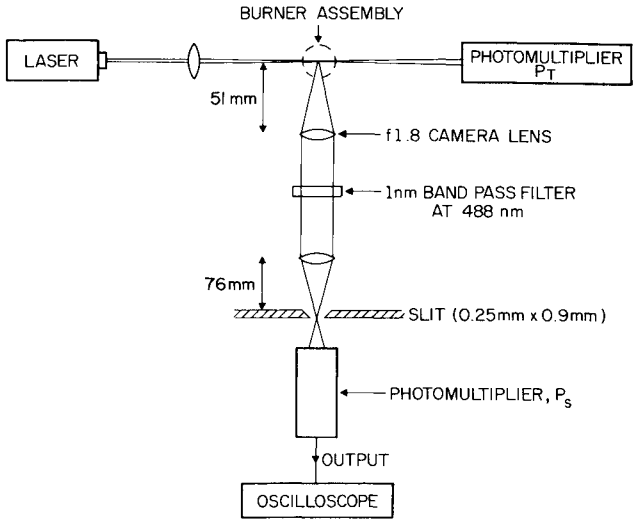

Fig. 1 Details of the optical system (plan view).

Molecular Rayleigh and Raman scattering are well suited to obtain local instantaneous measurement of individual gas densities. They are particularly advantageous in the study of hostile or inaccessible environments, and also when the presence of a physical probe would cause an unacceptable perturbation to the system under study. In this Note we discuss briefly the range of systems where Rayleigh rather than Raman scattering is of the greatest use and describe its successful application to the study of lifted jet diffusion flames. This particular regime has been chosen since we have recently shown,[1] using a gated mass-spectrometric probe technique, that the flow is dominated by the phenomenon of "unmixedness."[2] This technique does, however, still produce a certain element of time-averaging as well as physically disturbing the flow. It is also limited to the study of exactly periodic flows. Up to the present, Rayleigh (elastic) scattering‡ from gas flows has received far less attention than Raman (inelastic) scattering. This is because the frequency shift of a Raman transition (usually the Q branch of a vibrational band) together with the intensity and a polarization-dependent ratio serves in most cases to identify uniquely the chemical species concerned. Thus Vibrational Raman (VR) scattering measurements can be used to determine the concentrations of individual species in a multicomponent system. In contrast, Rayleigh scattering measurements can provide, at any instant, only a single intensity and a polarization ratio, no matter how many species are present, and thus their usefulness is restricted to simpler systems. However, the scattering cross sections for VR transitions are typically three orders of magnitude lower than the corresponding Rayleigh cross sections. Thus, with currently available light sources, measurements of transient concentrations by VR scattering (at atmospheric pressure) cannot be obtained with a time resolution better than about 1 sec. This is inadequate for studying the time-dependence of concentrations in turbulent flows, whereas a time resolution of better than 1 msec, which can be obtained from Rayleigh scattering measurements, is quite sufficient to resolve much of the turbulence structure.

General Principles

Molecular Rayleigh scattering is characterized by two variables, commonly chosen to be a total scattering cross section and the eccentricity of the polarizability ellipsoid. In this work, however, we choose the variables to be the differential scattering

‡ Throughout this paper Rayleigh scattering intensities and cross sections are assumed to include contributions from the entire rotational Raman spectrum and not just the transitions with $\Delta J = 0$.

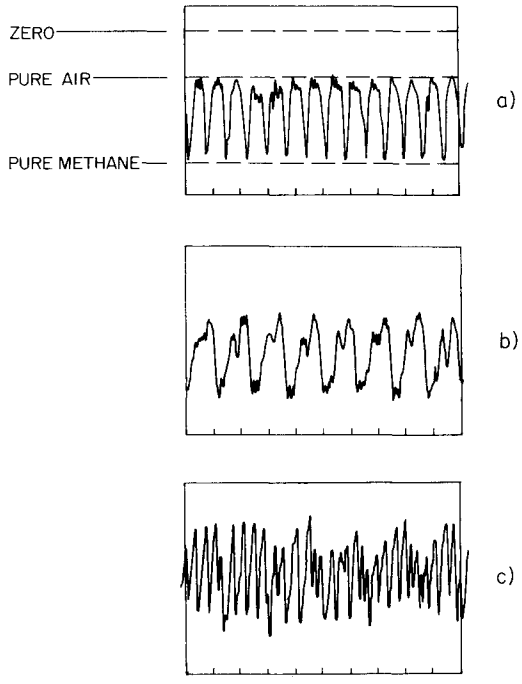

Fig. 2 a) Centerline concentration fluctuations showing virtually 100% unmixedness (2 msec/cm, 0.1 v/cm, $f = 600$ Hz, wavelength = 1.9 cm). b) Methane fluctuations in the shear layer showing stratified structure of vortex rings (1 msec/cm, 0.1 v/cm, $f = 900$ Hz, wavelength = 1.1 cm). Methane fluctuations in the absence of modulation (1 msec/cm, 50 mv/cm).

cross sections σ_e and σ_r which relate to light scattered perpendicular to a linearly polarized incident beam and scattered parallel and perpendicular, respectively, to the direction of this polarization.[3] For a mixture, at temperature T, of j gaseous species with absolute concentrations C_i (molecules/cm^3), and with corresponding differential scattering cross sections σ_{ei} and σ_{ri}, the corresponding fluxes are given by

$$I_e = k \sum_{i=1}^{J} C_i \sigma_{ei}(T); \quad I_r = k \sum_{i=1}^{J} C_i \sigma_{ri}(T) \tag{1}$$

where k is a constant dependent on the detection system and is proportional to the power of the incident beam. The cross sections are only weakly dependent upon temperature as the following analysis shows, and may in many applications be assumed constant.

An estimate of the change in Rayleigh scattering cross sections of a given species with temperature can be obtained by assuming that the cross section varies as the square of the molecular polarizability and hence as the square of the molecular volume.[3] The effective volume of a molecule increases with temperature because of the increasing population of the more energetic rotational states and of excited vibrational states. The centrifugal forces resulting from the increased rotational energy stretches chemical bonds in the molecule and in excited vibrational states the effective volume in general exceeds that of the ground vibrational state even in the harmonic oscillator approximation. As an example, the estimated relative change in cross section of a methane molecule with temperature, $(\partial\sigma_{CH_4}/\partial T)/\sigma_{CH_4}$, due to centrifugal forces is approximately constant at 2×10^{-5} per K so that the cross section is only 4% more at 2000 K than its value at 300 K. The vibrational effect in methane is likely to be much less than the stretching due to centrifugal forces because the molecule does not change its over-all size, to a first approximation, during a vibrational oscillation of either the C-H deformation or the asymmetric C-H stretch, whilst for the symmetric C-H stretch the population of the first excited state is only about 12% even at 2000 K.

In contrast, the Rayleigh scattering intensity from a given volume of methane or any other gas at constant pressure varies as the density. Thus the relative change in intensity with temperature is given by $(\partial\rho/\partial T)/\rho$, i.e., $-T^{-1}$, and the intensity thus falls by a factor of 6.7 as the temperature increases from 300 K to 2000 K. For most simple molecules, the variation in cross section with temperature will result primarily from centrifugal stretching but is not likely to exceed significantly that for methane, particularly for strongly bonded molecules such as O_2, N_2, CO, and CO_2.

Because the change in cross section with temperature is so much less than the corresponding density change, Eq. (1) may be solved for several different regimes.

For example, in a lifted jet diffusion flame much of the region below the flame is approximately isothermal and its composition may be expressed in the general form

$$xF + (1-x)(\tfrac{1}{5}O_2 + \tfrac{4}{5}N_2) \tag{2}$$

where F is the fuel species. In this simple mixing problem the determination of either I_e or I_r is thus sufficient to determine x, provided that (σ_e fuel/σ_e air) or (σ_r fuel/σ_r air) $\neq 1$.

For nonisothermal flows Eq. (1) may be used to measure temperatures in a fully mixed gas. This again only requires the measurement of either I_e or I_r, although the temperature-dependence of the corresponding cross section, though small, is in principle required and may either be estimated theoretically or be obtained experimentally by measuring the light scattered by the gas at a series of temperatures spanning the range of interest. The measurement of temperature by a method that causes no physical disturbance to the flowfield is particularly attractive, for example, in the study of heat transfer in both steady and nonsteady boundary layers.

If both I_e and I_r are measured, a two-component system may also be analyzed under conditions of variable temperature but constant pressure, provided again that the temperature-dependence of the various cross sections is known. This type of system would include, for example, the convective instability of stratified flows.

We now illustrate the potential of the technique with experimental results obtained from one of the abovementioned systems, namely the unburned region of a lifted jet diffusion flame. Here it is only necessary to measure either I_e or I_r, and as the perpendicular cross sections, σ_{ri}, are very much greater than the parallel cross sections, σ_{ei}, only I_r was measured.

Experimental Details and Results

The optical system is shown in plan view in Fig. 1. The linearly polarized beam (900 mw, $\lambda = 488$ nm) from a Coherent Radiation 52G argon-ion laser was focused with a 280 mm focal

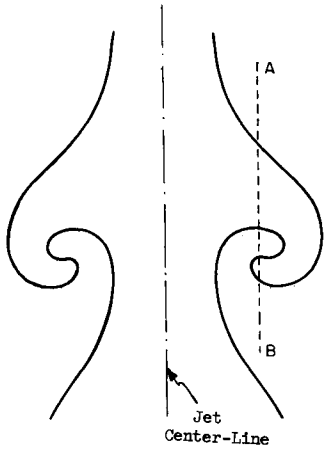

Fig. 3 Typical vortex structure in an axisymmetric jet. AB represents the locus of the scattering center relative to the moving vortex, and shows the origin of the double peak structure in Fig. 2b.

length lens to give a narrow beam of almost constant diameter (0.15 mm approx.) perpendicular to the gas flow. The light scattered perpendicular to the flow, to the incident beam and to the direction of polarization of the beam was collected and collimated by an f1.8 camera lens. This collimated beam passed through a 1 nm band-pass interference filter and was then focused at a slit so as to eliminate stray light before finally being detected by a photomultiplier. The slit also served to determine the length of the sample volume at 0.64 mm. For each experiment the laser power, which was monitored by the photomultiplier, P_T, in Fig. 1, and the gain of the photomultiplier, P_S, were held constant.

The burner assembly consisted of an axisymmetric fuel jet, formed by a quartz nozzle with throat 2.4 mm, and a concentric dust shield, 100-mm-diam, through which a low-velocity airstream was passed. The nozzle was mounted on a settling chamber containing a 50-mm-diam loudspeaker which could be used to impress a small periodic velocity modulation on the jet.[1] Both the fuel (methane) and air flows were filtered using a 25-mm-diam cellulose nitrate filter (pore size, 0.6 μm) to remove dust. This was necessary because the light scattered by a single dust particle can exceed, by more than an order of magnitude, that scattered by all the molecules in the beam. We were able to remove dust particles from the fuel flow completely (less than one particle every 5 min on average) but were less successful with the air supply (approximately one particle every two seconds). However, this latter level was sufficiently low for the purposes of these experiments.

The system was first calibrated by measuring the intensity of the light scattered from pure methane and pure air. The relative proportions (by mole) of methane and air in the sample volume may then be simply obtained by linear interpolation between the pure air and pure methane values. Some fluctuation of the species signals must be expected from photon and photomultiplier noise but secondary contributions also arise from fluctuations in the laser power and possibly from dust particles in the laser beam outside the sample volume. However, since such fluctuations were small in comparison with the concentration-dependent fluctuations no attempt was made to minimize them.

The limiting frequency response, ν_{max} Hz, of the Rayleigh scattering detection system depends on the signal-to-noise ratio in the measurement of the number of photons detected during the time interval, ν_{max}^{-1} sec, and consequently ν_{max} varies as the square root of the photon flux reaching the detecting photomultiplier. In the experiments reported here this flux exceeds 10^8 photons/sec so that, for a photocathode quantum efficiency of 10%, the limiting frequency response exceeds 10 kHz.

A signal from a point on the centerline of the initial mixing region of a jet modulated at 600 Hz using the settling chamber loudspeaker is shown in Fig. 2a and it is significant that the wave form shows 100% unmixedness. The time response of the scattering system is sufficient to identify regions of 100% unmixedness at high modulation frequencies which was not possible with the gated sampling probe.[1] The stratified structure of the vortex rings which dominate the intense shear region of jet flows (shown schematically in Fig. 3) is resolved as double peaks in the signal in Fig. 2b (modulated at 900 Hz).

Finally, a typical oscillogram of scattering from a methane jet in the absence of modulation, and at a higher flowrate, is shown in Fig. 2c. Again vortex shedding is clearly apparent from the periodicity in the signal although less well-defined than previously, since the phase locking of the loudspeaker is no longer present. In order to follow fluctuations of such rapidity, Rayleigh scattering represents the only viable measuring technique.

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Prediction of Yawing Force at High Angle of Attack

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Nomenclature

C_N = normal force coefficient = Normal Force/$q_\infty \pi r^2$
C_y = yaw force coefficient = Yaw Force/$q_\infty \pi r^2$
D = diameter of body of revolution
k = number of vortices in the flow
j = pertaining to the jth vortex
M = freestream Mach number
q_∞ = freestream dynamic pressure
r = radius of body of revolution
r_0 = reference radius
t = time
U = freestream velocity
w = dimensionless crossflow plane velocity = $(\overline{d\phi}/d\zeta)/U \sin\alpha$
w_j = dimensionless crossflow plane velocity at the location of the jth vortex. The influence of the jth vortex is neglected
x, y, z = Cartesian coordinates (see Fig. 1)
α = angle of attack
Γ = vortex circulation
ζ = complex crossflow plane coordinate = $y+iz$
ζ_{0_1}, ζ_{0_2} = crossflow plane separation points (see Fig. 1)
$\theta_{0_1}, \theta_{0_2}$ = crossflow plane separation angles (see Fig. 1)
ρ = density
λ = dimensionless circulation = $\Gamma/2\pi r U \sin\alpha$
ϕ = crossflow velocity potential
ϕ_N = nose roll angle

Introduction

At high angles of attack the leeward flowfield about a body of revolution separates and rolls up to form vortices. For relatively small angles, two symmetric vortices are formed. At larger angles, wake vortices grow unsymmetrically and are shed from the leeward side, as illustrated in Fig. 1. The resulting flowfield asymmetries induce a yawing force and moment. Simple engineering calculations can be used to determine normal forces and moments,[1-3] but a more detailed flow description must be employed if yaw forces are to be predicted.

In this Note, an approximate flowfield description is developed which is an extension of the model originally proposed by Bryson[4] and modified by Schindel.[5] The current formulation is applicable to bodies of revolution and models the flow using the impulsive flow analogy with viscous effects being simulated using point vortices. The symmetric wake assumption present in preceding studies is relaxed and growing vortices near the missile, as well as those shed into the wake, are taken into the account.

Problem Formulation

The velocity potential in the crossflow plane is formed by superimposing point vortex potentials on the potential function for flow over a cylinder. An additional source term is included to account for the changing body radius

$$\phi(\zeta) = U \sin\alpha \left\{ -\left(\zeta - \frac{r^2}{\zeta}\right) i - ir \sum_{j=1}^{k} \lambda_j \ln\left[\frac{(\zeta-\zeta_j)}{[\zeta-(r^2/\zeta_j)]}\right] + \frac{r}{\tan\alpha}\frac{dr}{dx}\ln(\zeta) \right\} \quad (1)$$

The motion of vortices in the crossflow plane is assumed to be determined by balancing the lift force on each vortex $[-i\rho\Gamma(d\zeta/dt - w_j U \sin\alpha)]$ with the force on its feeding sheet

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